Math 0413: Introduction to Theoretical Mathematics

Class Times: MTuWTh 6:00PM–7:25PM (158 Benedum Hall).

Professor: Bogdan Ion, email: bion@pitt.edu, office: 423 Thackeray Hall, phone: 624-8343.

Recitation instructor: Cezar Lupu, email: cel47@pitt.edu.

Office Hours: MTuWTh 5:30–6:00PM or by appointment in 423 Thackeray Hall.

Web Site: Announcements, handouts, and other resources will be made available at http://www.pitt.edu/~bion/0413.html

Textbook: Basic Analysis. Custom Pitt edition by Jiří Lebl. The book is freely available at http://calculus.math.pitt.edu/books/pittanal.pdf. Printed copies are available from the Pitt Book Center. The course covers (in this order) the material in Appendix A, Introduction, Appendix B, Chapter 1, and Chapter 2. However, at least for the topics outside Chapter 1 and 2, the best resource for the material will be your class notes and the notes posted on the course web page. A reference that I find to be considerably more useful than the textbook for the first half of the course is Introduction to Mathematical Thinking by Keith Devlin. 2012.

Prerequisites: Math 0230.

Grading Policy: The final grade will be computed as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight</th>
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<tbody>
<tr>
<td>Homework</td>
<td>25%</td>
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<tr>
<td>Final project</td>
<td>25%</td>
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<tr>
<td>Exam 1</td>
<td>10%</td>
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<tr>
<td>Exam 2</td>
<td>10%</td>
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<tr>
<td>Exam 3</td>
<td>30%</td>
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The grades will not be curved. Roughly, the following grading scale will apply:

A: 90-100, B: 80-90, C: 70-80, D: 55-70.

Course event calendar:

- Homework 1: Monday, May 18
- Exam 1: Thursday, May 21
- Homework 2: Tuesday, May 26
- Project topic: Friday, May 29
- Homework 3: Monday, June 1
- Exam 2: Thursday, June 4
- Project outline due: Friday, June 5
- Homework 4: Monday, June 8
- Project draft due: Friday, June 12
- Homework 5: Monday, June 15
- Exam 3: Thursday, June 18
- Project draft due: Friday, June 19

Other Important dates:

- Add/drop period ends: Wednesday, May 13
- Memorial Day: Monday, May 25
- Last day for monitored withdrawals: Friday, June 5
- Last Day of Classes: Thursday, June 18

Exams: There are no make-up exams. You may not be excused from an exam, except in the event of a documented emergency, and then only with my permission. Note that travel is not a sufficient excuse to have an exam scheduled on a different day.

Perhaps unlike previous math courses there will be an emphasis on the detailed understanding of the theorems and the proofs that will be studied throughout the semester. A part of every exam will ask for the statement and proof of one of the main theorems in the course. A list of these main results is posted on the course web page.
Writing: This is a writing intensive course. This means, in particular, that there will be regular writing assignments throughout the semester, generally increasing in complexity and length, totaling a minimum of 20-25 pages. The writing assignments will consist of homework assignments and a final project. Since the writing is specific to the discipline, the documents have to be prepared in \TeX, which is the standard typesetting language used in mathematics and other scientific fields. Detailed information on how to get started and how to create documents using \TeX is available on the course web page.

There is dramatic shift in what the subject matter is generally concerned with. Previous mathematics courses had largely a computational emphasis, most questions considered had a numerical answer, and the material focused to large extent on the tools and methods employed in computations, perhaps with some informal justification. Starting with this course the emphasis is on the truth value (i.e. true/false) of statements. The procedure by which the truth value is determined is formal/rigid, in the sense that in the arguments we are only allowed to use true statements certain procedures that produce true statements (the inference rules of first-order logic). The writing will necessarily reflect this shift and one of the goals of the course is to learn how to write mathematical arguments following the modern methodology.

Homework: The homework consists of 5 assignments which will have to be turned in during recitation on the dates specified above. The homework assignments are posted on the course web page. Late homework is not accepted. Collaboration is allowed, but each student must write up and submit his own solutions and make sure to understand thoroughly the results obtained jointly. In order to be graded all writings must be coherent, typed in \TeX, and submitted in the following format:

1. The problems should be printed on 8.5”x11” paper in the assigned order with the assignment and your name written at the top of the first page.

2. For each assigned problem, first copy the statement of problem, then give its solution. No credit is given for a solution to a misstated assigned problem.

3. Write effectively, with due attention to organization and logical progression of ideas. Each solution should be complete and appropriately supported - by relevant observations, argumentation, drawings etc. - but direct and to the point. The paper should be easily readable by your peers.

Final project: The final project is devoted to the development of a mathematical topic that you will have to choose in consultation with me. A (non-exhaustive) list of possible topics for the final project is posted on the course web page but other topics are also possible. The length of the final document will likely fall within 5-7 pages. The paper will consist of the mathematical exposition of one or several important results relevant to the chosen topic and will be similar in style and content to a section of the textbook. Precise references will be indicated.

Course description: Between 1890 and 1930 mathematics underwent a modernist transformation in methodology, which shifted from the so-called scientific method, that relies on physical intuition and for which validity depends on comparison with external reality (explanations are valid as long as they are thoroughly tested), to the axiomatic method, that relies on precise definitions and on internal criteria for validity (rules and procedures that, when followed, produce completely reliable conclusions). The new methodology arose as a result of thousands of years of experimentation and allows modern mathematics to go considerably beyond what was previously accessible through the scientific method. Besides setting the field on a firm foundation it also led to surprising or counter-intuitive facts such as the discovery of surfaces that cannot be embedded in 3-dimensional space, the existence of functions that are everywhere continuous but nowhere differentiable (Weierstrass), the discovery of curves that fill up space (Peano), and the understanding that an interval and a cube contain the same number of points (Cantor), to name just a few.

The course is an introduction to the modern methodology and forms the foundation on which all subsequent advanced courses in core mathematics will rest upon. The topics discussed will include logic,
set theory, and the theoretical treatment of the real numbers culminating with the Bolzano-Weierstrass theorem. Classwork and homework will concentrate on assimilation of the modern methodology through the writing and understanding of proofs of theorems centered on these topics.

**How to succeed:** The main purpose of all the work you will be doing throughout the semester is to achieve the level of mastery required to succeed in subsequent math courses and to build up your mathematical writing skills. The main difficulty of the material consists in the novelty of the manner of thinking which departs from the mathematics with a computational emphasis to which you already have had lengthy exposure to. Your experience will be more akin to learning a new language or a new computer programming language. Just as in these situations, the best approach to assimilating the material is through immersion. Spending consistent time with the material, especially at the beginning, will be particularly beneficial. There will be some tension between intuition and the new methodology (e.g. what might be intuitively clear might be very far from being validated with respect to the new methodology) and confusion is to be expected, but perseverance is the key to overcoming it.

The course is designed to be accessible to students that are seeing proofs for the first time. It allows students to benefit from a fresh start but it is important to invest time early on learning how to think about proofs and counterexamples and adapting old techniques to new situations rather than chugging through computational problems similar to the examples in the textbook and the lecture. Lectures and recitations are not a spectator sport but rather your opportunity to build up your new skills. Recitations, in particular, are indispensable and participation and attendance are a must. Help is available in the lecture, recitation, office hours, and the MAC center.

**Accommodations and schedule conflicts:**
If you have a disability for which you are or may be requesting an accommodation, you are encouraged to contact both me and Disability Resources and Services, 216 William Pitt Union, (412)648-7890/(412)383-7355 (TTY), before the end of the second week of classes. Similarly, students who have any conflicts (including religious observances) with the scheduled examination dates should notify me before the end of the second week of classes.

**University of Pittsburgh e-mail Policy 09-10-01:**
Each student is issued a University e-mail address (username@pitt.edu) upon admittance. This e-mail address may be used by the University for official communication with students. Students are expected to read e-mail sent to this account on a regular basis. Failure to read and react to University communications in a timely manner does not absolve the student from knowing and complying with the content of the communications. The University provides an e-mail forwarding service that allows students to read their e-mail via other service providers (e.g., Hotmail, AOL, Yahoo). Students that choose to forward their e-mail from their pitt.edu address to another address do so at their own risk. If e-mail is lost as a result of forwarding, it does not absolve the student from responding to official communications sent to their University e-mail address. The link to this policy is located at: [http://www.bc.pitt.edu/policies/policy/09/09-10-01.html](http://www.bc.pitt.edu/policies/policy/09/09-10-01.html). Instructions on how to forward e-mail messages are at: [http://www.technology.pitt.edu/email-accounts/email/imap/imap-forward.html](http://www.technology.pitt.edu/email-accounts/email/imap/imap-forward.html)