PROBLEMS AND SOLUTIONS

Edited by Gerald A. Edgar, Doug Hensley, Douglas B. West

Proposed problems and solutions should be sent in duplicate to the MONTHLY problems address on the inside front cover. Submitted solutions should arrive at that address before June 30, 2008. Additional information, such as generalizations and references, is welcome. The problem number and the solver’s name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.

PROBLEMS

11341. Proposed by Cezar Lupu, University of Bucharest, Bucharest, Romania (student), and Tudorel Lupu, Decebal High School, Constanza, Romania. Consider an acute triangle with side-lengths $a$, $b$, and $c$, with inradius $r$ and semiperimeter $p$. Show that

$$(1 - \cos A)(1 - \cos B)(1 - \cos C) \geq \cos A \cos B \cos C \left(2 - \frac{3\sqrt{3}r}{p}\right).$$

11342. Proposed by Luis H. Gallardo, University of Brest, Brest, France. Let $p$ be a prime and let $\mathbb{F}_p$ be a finite field of characteristic $p$, where $q$ is a power of $p$. Let $n$ be a divisor of $q - 1$. With the natural mapping of $\mathbb{Z}$ onto $\mathbb{F}_p$ and embedding of $\mathbb{F}_p$ in $\mathbb{F}_q$, show that $(-1)^{(n+2)(n-1)/2} n^a$ is a square in $\mathbb{F}_q$.

11343. Proposed by David Beckwith, Sag Harbor, NY. Show that when $n$ is a positive integer,

$$\sum_{k \geq 0} \binom{n}{k} \binom{2k}{k} = \sum_{k \geq 0} \binom{n}{2k} \binom{2k}{k} 3^{n-2k}.$$

11344. Proposed by Albert Stadler, Meilen, Switzerland. Let $\mu$ be the Möbius $\mu$ function of number theory. Show that if $n$ is a positive integer and $n > 1$, then

$$\sum_{j=1}^{n} \mu(j) = - \sum_{j=1}^{[n/2]} j \sum_{k=[(n+1)/(2j+1)]} \mu(k).$$

11345. Proposed by Roger Cuculièr, France. Find all nondecreasing functions $f$ from $\mathbb{R}$ to $\mathbb{R}$ such that $f(x + f(y)) = f(f(x)) + f(y)$ for all real $x$ and $y$.

11346. Proposed by Christopher Hillar, Texas A&M University, College Station, TX, and Lionel Levine, University of California, Berkeley, CA. Let $n$ be an integer greater