Problem 1. Solve the following differential equations/initial value problems:

a) \( y' = \frac{y}{x}, \quad y(1) = -2 \)
b) \( y' = xy + y \)
c) \( x^2y' = y \log y - y' \)
d) \( y' = e^{x+y}, \quad y(0) = 0 \)
e) \( y' = \frac{\sin x}{y}, \quad y(\pi/2) = 1 \)
f) \( xy' + 2y = \sin x, \quad y(\pi/2) = 0 \)
g) \( y' = y + 2xe^{2x}, \quad y(0) = 3 \)
h) \( tx' = 4x + t^4 \)
i) \( (1 + x^3)y' = 3x^2y + x^2 + x^5 \)
j) \( (y^2 - xy)dx + x^2dy = 0, \quad \mu(x, y) = \frac{1}{xy} \)
k) \( (x^2 + y^2 - x)dx - ydy = 0 \quad \mu(x, y) = \frac{1}{x^2+y^2} \)
l) \( (u + v)du + (u - v)dv = 0 \)
m) \( \frac{2u}{x^2+y^2}du + \frac{2v}{x^2+y^2}dv = 0 \)
n) \( (x^2y^2 - 1)ydx + (1 + x^2y^2)xdy = 0, \quad \mu(x, y) = \frac{1}{xy} \)

Problem 2. a) Suppose a cold beer at 40°F is placed into a warm room at 70°F. Suppose that 10 minutes later, the temperature of the beer is 48°F. Use Newton’s law of cooling to find temperature 25 minutes after the beer was placed into the room.

b) Referring to part a), suppose that a 50°F bottle of beer is discovered on a kitchen counter in a 70°F room. Ten minutes later, the bottle is 60°F. If the refrigerator is kept at 40°F, how long had the bottle of beer been sitting on the counter when it was first discovered?

Problem 3. a) A murder victim is discovered at midnight and the temperature of the body is recorded at 31°C. One hour later, the temperature of the body is recorded at 29°C. Assume that the surrounding air temperature remains constant at 21°C. Use Newton’s law of cooling to calculate the victim’s time of death.

b) Suppose we discover another murder victim at midnight with body temperature of 31°C. However, this time the air temperature at midnight is 0°C, and is falling at a constant rate of 1°C per hour. At what time did the victim die?

Problem 4. When a cake is removed from a baking oven its temperature is measured at 300°F. Three minutes later its temperature is 200°F. How long will it take to cool off to a room of 70°F?
Problem 5. a) Suppose that \((xy-1)dx+(x^2-xy)dy = 0\) has an integrant factor that is a function of \(x\) alone (i.e. \(\mu = \mu(x)\)). Find the integrant factor and use it to solve the differential equation.

b) Suppose that \(2ydx + (x+y)dy = 0\) has an integrant factor that is a function of \(y\) alone (i.e. \(\mu = \mu(y)\)). Find the integrant factor and use it to solve the differential equation.