1. Suppose that $f(x)$ is defined on $[-1, 1]$ and is $C^3$, i.e. the third order derivative $f^{(3)}(x)$ is continuous on $[0, 1]$. Determine whether the following series is convergent or divergent

$$\sum_{n=1}^{\infty} \left( n \left( f \left( \frac{1}{n} \right) - f \left( -\frac{1}{n} \right) \right) - 2f'(0) \right)$$

*Hint: Use Taylor Theorem.*
2. Let $D$ be a non-empty, open and convex subset of $\mathbb{R}^m$ and $f : D \to \mathbb{R}^n$ is such that there exists $\alpha > 1$ and $L > 0$ with

$$\|f(x) - f(y)\| \leq L \cdot \|x - y\|^\alpha$$

for all $x, y \in D$. Show that $f$ is constant.

*Hint: If $f$ is not constant, then there exists $c \in \mathbb{R}^m$ such that the total derivative $f'(c)$ is not zero. Next reduce to the case $n = 1$ and use mean value theorem.*
3. Decide whether the following function is differentiable at (0,0):

\[
f(x, y) = \begin{cases} 
  \frac{x^2(y^4 + 2x)}{x^2 + y^6} & \text{if } (x, y) \neq (0, 0); \\
  0 & \text{if } (x, y) = (0, 0). 
\end{cases}
\]