Math 0290
Summer 2014
Final Exam
8/1/2014
Time Limit: 3 Hours
Class: Differential Equations for Engineers

This exam contains 11 pages (including this cover page) and 10 questions. Total of points is 30.
This is an open book and notes exam. Calculators are allowed. Show all your work (no work = no credit). Write neatly. Simplify your answers.

Grade Table (for teacher use only)

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Good luck !!!
1. (3 points) Solve the following initial value problems. Mention a type of the differential equation:

a) \( \frac{y'}{3} = x^2y \), \( y(0) = -8 \)

b) \( t \frac{dx}{dt} = 4x + t^4 \), \( x(1) = 5 \)

c) \( y'' - 4y' = 2e^{4t} \), \( y(0) = -3 \) and \( y'(0) = 11 \).
2. (3 points) Determine whether the given equation is exact. If it is exact, solve it:

\[(\sin y - y \sin x)dx + (\cos x + x \cos y - y)dy = 0.\]
3. (3 points) When a cake is removed from a baking oven its temperature is measured at $300^\circ$ F. Three minutes later its temperature is $200^\circ$. How long will it take to cool off to a room of $70^\circ$ F?
4. (3 points) Find the general solution to the equation

\[ \frac{1}{4} y'' + y' + y = t^2 - 2t. \]
5. (3 points) Find the Laplace transform of the function

\[ g(t) = \begin{cases} 
3t, & 0 \leq t < 2 \\
4, & t \geq 2 
\end{cases} \]
6. (3 points) Suppose that \( f \) is a continuous function for \( t \geq 0 \) and is of exponential order.

a) If \( f(t) \leftrightarrow F(s) \) is a transform pair, prove that

\[
\mathcal{L} \left( \int_0^t f(\tau) d\tau \right) = \frac{F(s)}{s}.
\]

b) Use the technique from a) to find

\[
\mathcal{L}^{-1} \left( \frac{1}{s(s^2 + 1)} \right).
\]
7. (3 points) Use the Laplace transform to solve the following initial value problem:

\[ y'' + 16y = \cos 4t, \quad y(0) = 0, \quad y'(0) = 8. \]
8. (3 points) Find the general solution to the system:

\[ y' = \begin{bmatrix} -3 & -6 \\ 0 & -1 \end{bmatrix} y \]
9. (3 points) Find the general solution to the following system:

\[ x' = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} x. \]
10. (3 points) Expand the given function in a Fourier cosine series valid on the interval $0 \leq x \leq \pi$. Calculate $a_0$ separately.

$$f(x) = x.$$