1. Prove that

\[
\lim_{n \to \infty} \left( \frac{1}{n + 1} + \frac{1}{n + 2} + \cdots + \frac{1}{n + n} \right) = \log 2.
\]
2. Prove that the series
\[ \sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2} \]
converges uniformly on \([0, 1]\).
3. Let $f$ be a $C^2$ function on $\mathbb{R}$. Suppose that $a > 0$ and $|f''(x)| \leq M$ for $x \in [-a, a]$. Prove that

$$\left| \int_{-a}^{a} f(x) \, dx \right| \leq 2a|f(0)| + \frac{a^3 M}{3}.$$
4. Let \( \{f_n\} \) be a uniformly bounded sequence of functions which are Riemann integrable on \([a, b]\), and put
\[
F_n(x) = \int_a^x f_n(t)\,dt \quad (a \leq x \leq b).
\]
Prove that there exists a subsequence \( \{F_{n_k}\} \) which converges uniformly on \([a, b]\).