CALCULUS III (MATH 0240), SUMMER 2016
HOMEWORK NO. 1

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Problem 1. Find a vector that is orthogonal to both vectors $\vec{i}-\vec{j}+\vec{k}$ and $\vec{i}-\vec{k}$.

Problem 2. Find the scalar (component) and vector projections of $\vec{b}=(5,-\sqrt{3},7)$ onto $\vec{a}=(-2,\sqrt{3},3)$.

Problem 3. Are vectors $\vec{a}=(1,2,-3)$, $\vec{b}=(4,1,-2)$, and $\vec{c}=(1,3,5)$ coplanar?

Problem 4. Find a symmetric equation of a line such that the cosines of angles between the linee of each coordinate axes are all $\frac{1}{\sqrt{3}}$.

Problem 5. Find the angle between the vectors $\vec{a}=(-3,4,2)$ and $\vec{b}=(3,1,7)$.

Problem 6. Find parametric equations of a line such that cosine of the angle between the line and the x-axis is $\frac{1}{3}$, cosines of angles between the line and the y-axis and z-axis are both $\frac{2}{3}$.

Problem 7. Find parametric equations and symmetric equations for the line intersection of the planes $x-2y+z=1$ and $x+z=0$.

Problem 8. Describe and sketch the surface $y^2+4z^2=4$.

Problem 9. Classify the surface $x^2+y^2+z^2-8x-6y-8z+24=0$.

Problem 10. Evaluate each of the following limits or demonstrate that it does not exist:

(i) $\lim\limits_{(x,y)\to(1,-1)} e^{-xy}\cos(x+y)$.

(ii) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$.

(iii) $\lim\limits_{(x,y)\to(0,0)} \dfrac{e^{-x^2-y^2}-1}{x^2+y^2}$.

(iv) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^4}$.

Problem 11. Find the indicated partial derivatives:

(i) $f(x,y)=\log(x+\sqrt{x^2+y^2})$; $\dfrac{\partial f}{\partial x}(3,4)=f_x(3,4)$.

(ii) $f(x,y) = \arctan\left(\frac{y}{x}\right)$; $\frac{\partial f}{\partial x}(2,3) = f_x(2,3)$.

Problem 12. (i) Find the gradient of $f(x,y) = \sin(y \log x)$.
(ii) Evaluate the gradient at the point $P(1,-2)$.
(iii) Find the rate of f at P in the direction of the vector $\vec{u} = \left(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)$.

Problem 13. Find the local minimum and maximum values and saddle point(s) of the following functions:

(i) $f(x,y) = x^2 + xy + y^2 + y$.
(ii) $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$.

Problem 14. Find the minimum and the maximum values of the function $f(x,y) = 4x^2 + 9y^2$ subject to the constraint $xy = 4$.

Problem 15. A cardboard box without a lid is to have a volume of 32000 cm^3. Find the dimensions that minimize the amount of cardboard used.