1. Let $A \in \mathfrak{M}_2(\mathbb{R})$ such that there exists $n$ such that $[A, B]^n = I_2$. Then, $n$ is even and $[A, B]^4 = I_2$.
2. Let $A, B, C, D \in \mathfrak{M}_2(\mathbb{R})$. Show that $[A, B][C, D] + [C, D][A, B]$ is a scaling of $I_2$.
3. Let $A, B \in \mathfrak{M}_2(\mathbb{R})$ such that $[A, B] = 0$ and $\det(A^2 + B^2) = 0$. Then $\det(A) = \det(B)$.
4. Let $A, B \in \mathfrak{M}_3(\mathbb{R})$ such that $\det(A) = \det(B) = \det(A - B) = \det(A + B) = 0$.
   Then, $\det(xA + yB) = 0$ for any $x, y \in \mathbb{R}$.
5. Let $A \in \mathfrak{M}_3(\mathbb{R})$ such that $\text{tr}(A) = \text{tr}(A^2) = 0$. Show that $\det(A^2 + I_3) = \det(A)^2 + 1$. 