Math 0120  
Examination #2  
Sample  

Name (Print) ___________________________  PeopleSoft# __________.  

Signature______________________________  Score___________  

TA (Circle one)  

Instructions:  

1. Clearly print your name, sign your name and enter your Peoplesoft identification number in the space above.  

2. There are 7 problems, each worth the specified number of points, for a total of 100 points. There is also an extra-credit problem worth up to 5 points.  

3. Please work each problem in the space provided. Extra space is available on the back of each exam sheet. Clearly identify the problem for which the space is required when using the backs of sheets.  

4. Show all calculations and display answers clearly. Unjustified answers will receive no credit.  

5. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.  

6. Calculators may not be used. All derivatives are to be found by learned methods of calculus.
1. (30 pts.) Find the derivatives of the following functions (you need not simplify):

(a) \( f(x) = e^{-x^3} - \frac{e^{2-x}}{\ln(1-x)} \)

(b) \( f(x) = (e^{\sqrt{x}})\ln x + \frac{4}{e^{x}} \)

(c) \( f(x) = \sqrt[3]{\ln(1-x^5)} - \frac{1}{(x-2)^4} \).

2. (5 pts.) The resale value \( R \) (in dollars) of a certain model car after \( t \) years is given by

\[ R(t) = 20,000 e^{-0.1t} \].

What are the instantaneous and relative rates of change (depreciation) the moment the car is sold \( (t = 0) \)?
3. (15 pts.) A poster is to have 2-inch margins at the top and bottom and 1-inch margins on the sides. The total area is to be 162 square inches. Find the dimensions that maximize the print area. (The area of a rectangle is given by $A=(\text{length})(\text{width})$.)
4. (20 pts.) \( f(x) = x^4 - 4x^3 = x^3(x - 4) \). \( f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3) \), and \( f''(x) = 12x^2 - 24x = 12x(x - 2) \).

(a) Find the critical numbers and the inflection points of \( f \).  
(b) Construct sign charts for the first and second derivatives.  
(c) Find all open intervals of increase and decrease and open intervals on which the graph is concave up and concave down.  
(d) Classify each critical point as a relative maximum, relative minimum or neither.  
(e) Sketch the graph of \( y = f(x) \) by hand, labeling only the relative extreme points and the inflection points (Note that \( f(-3) = 189, f(-2) = 48, f(0) = 0, f(2) = -16, \) and \( f(3) = -27 \)).
5. (10 pts.) A cherry tree will yield 100 pounds of peaches now, which will sell for 60 cents a pound. Each week that the farmer waits will increase the yield by 5 pounds, but the selling price will decrease by 2 cents per pound. How long should the farmer wait to pick the fruit in order to maximize his revenue?

6. (10 pts.) A demand function is given by $D(p) = 12,000 - 10p^2$. Find the elasticity of demand, $E(p)$, at $p = 20$. Determine whether the demand is elastic, inelastic, or unitary at $p = 20$. Should the price be raised, lowered, or left the same, in order to increase revenue?
7. (10 pts.) \( xy^2 = 2y + x^2 \). Find \( \frac{dy}{dx} \) and an equation of the tangent line at \( x = 2 \) and \( y = -1 \).

Extra credit (5 pts.) If \( \log_a 2 = W, \quad \log_a 3 = X, \quad \log_a 5 = Y, \quad \text{and} \quad \log_a 7 = Z, \) find \( \log_a \left( \frac{9}{14} \right) \).