Problem 1. Given nine points inside the unit square, prove that some three of them form a triangle whose area does not exceed $\frac{1}{8}$.

Problem 2. Let $(x_n)_{n \geq 0}$ be a sequence defined by $x_0 > 0$ and $x_{n+1} = x_n + \frac{1}{\sqrt{x_n}}, n \geq 0$. Evaluate

$$\lim_{n \to \infty} \frac{x_n^2}{n^3}.$$

Problem 3. Let $A \in M_2(\mathbb{R})$ such that $\text{Tr}(A) > 2$. Show that $A^n \neq I_2$ for all $n \geq 1$.

Problem 4. Find the smallest positive integer $n$ such that for every integer $m$ with $0 < m < 1993$, there exists an integer $k$ for which

$$\frac{m}{1993} < \frac{k}{n} < \frac{m+1}{1994}.$$  

Problem 5. Let $f : [0, 1] \to \mathbb{R}$ be a differentiable function with continuous derivative such that $f(0) = f(1) = 0$. Show that

$$\int_0^1 (f'(x))^2 dx \geq 12 \left( \int_0^1 f(x) dx \right)^2.$$

Problem 6. Let $A \in M_n(\mathbb{Z})$. Show that $\det(A^4 + I_n) \neq 13$. 

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