Problem 1. Given 7 distinct positive integers that add up to 100, prove that some three of them add up to at least 50.

Problem 2. Find all real numbers $x$ such that $2^x + 2^{1/x} = 4$.

Problem 3. Let $A, B \in M_n(\mathbb{R})$, $A \neq B$ such that $A^3 = B^3$ and $A^2 B = B^2 A$. Is it possible for the matrix $A^2 + B^2$ to be invertible?

Problem 4. Let $(a_n)_{n \geq 1}$ be a sequence defined by

$$a_n = \int_0^n \log(1 + e^{-x})dx.$$  

Show that the sequence converges and its limit is in the interval $\left[\frac{3}{4}, 1\right]$.

Problem 5. Let $A, B \in M_2(\mathbb{C})$ such that $AB = BA$ and

$$\det(A + B) = \det(A + 2B) = \det(A + 3B) = 1.$$  

Show that $B^2 = O_2$.

Problem 6. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function such that $f(1) = 0$. Show that there exists $c \in (0, 1)$ such that

$$f(c) = \int_0^c f(x)dx.$$