**Problem 1.** Find the following limits:

(i) \( \lim_{n \to \infty} n \left( \log 2 - \sum_{k=1}^{n} \frac{1}{n+k} \right) \)

(ii) \( \lim_{n \to \infty} \left( \frac{1}{n} \sum_{k=1}^{n} \sqrt{1 + \frac{1}{n+k}} \right)^n \).

**Problem 2.** Let \((a_n)_{n \geq 1}\) be a sequence of positive integers such that \( \sum_{n \geq 1} a_n^3 \) converges. Show that the series \( \sum_{n \geq 1} \frac{a_n}{n} \) converges also.

**Problem 3.** Does there exist a sequence \((a_n)_{n \geq 1}\) of positive reals such that the series \( \sum_{n \geq 1} \frac{1}{a_n} \) converges and \( \sum_{k=1}^{n} a_k \leq n^2 \) for all \( n \geq 1 \)?

**Problem 4.** Show that the series \( \sum_{n=1}^{\infty} \frac{1}{p_n} \) is convergent, where \( p_1, p_2, \ldots, p_n \) are positive real numbers, then the series

\[
\sum_{n=1}^{\infty} \frac{n^2}{(p_1 + p_2 + \ldots + p_n)^2} \cdot p_n
\]

is also convergent.

Putnam B3, 1966

**Problem 5.** Determine, with proof, whether the series

\[
\sum_{n=1}^{\infty} \frac{1}{n^{1.8 + \sin n}}
\]

converges or diverges.
Problem 6. Let $k$ be an integer greater than 1. Suppose that $a_0 > 0$, and define the sequence $(a_n)_{n \geq 0}$,

$$a_{n+1} = a_n + \frac{1}{\sqrt[n]{a_n}}, n \geq 0.$$ 

Evaluate

$$\lim_{n \to \infty} \frac{a_{n+1}^k}{n^k}.$$ 

Putnam B6, 2006

Problem 7. Let $(a_n)_{n \geq 1}$ be a sequence such that $x_n = \sum_{k=1}^{n} a_k^2$ converges, while $y_n = \sum_{k=1}^{n} a_k$ is unbounded. Show that the sequence $b_n = \{y_n\}$ diverges. Here $\{x\}$ is the fractional part of $x$.

Romanian National Olympiad, 1998