Problem 1. Find all continuous functions \( f : [0, 1] \to \mathbb{R} \) such that
\[
\int_0^1 f(x)(x - f(x))\,dx = \frac{1}{12}.
\]

Problem 2. Find all continuous functions \( f : \mathbb{R} \to [1, \infty) \) for which there exists \( a \in \mathbb{R} \) and a positive integer \( k \) such that
\[
f(x)f(2x)\ldots f(nx) \leq an^k,
\]
for every real number \( x \) and positive integer \( n \).

Romanian National Olympiad, 1999

Problem 3. Let \( f : [0, 1] \to \mathbb{R} \) be a continuous function such that \( \int_0^1 f(x)\,dx = 0 \). Show that there exists \( c \in (0, 1) \) such that
\[
f(c) = \int_0^c f(x)\,dx.
\]

Gazeta Matematică, 1992

Problem 4. Find
\[
\lim_{n \to \infty} \frac{1}{n} \int_0^n \frac{x \log(1 + x/n)}{1 + x}\,dx.
\]

American Mathematical Monthly, 2006

Problem 5. Let \( f : [0, 1] \to \mathbb{R} \) be a continuous function, differentiable on \((0, 1)\), with the property that there exists \( \alpha \in (0, 1) \) such that \( \int_0^\alpha f(x)\,dx = 0 \). Prove that
\[
\left| \int_0^1 f(x)\,dx \right| \leq \frac{1 - a}{2} \sup_{x \in (0,1)} |f'(x)|.
\]

Romanian National Olympiad, 1983

Problem 6. Let \( f : [0, 1] \to \mathbb{R} \) be an integrable function such that
\[ \int_0^1 f(x) \, dx = \int_0^1 x f(x) \, dx = 1. \]

Show that
\[ \int_0^1 f^2(x) \, dx \geq 4. \]

Romanian National Olympiad, 2004

**Problem 7.** Evaluate
\[ \int_0^1 \frac{\log(1 + x)}{1 + x^2} \, dx. \]

Putnam A5, 2005

**Problem 8.** Let \( f, g : \mathbb{R} \to \mathbb{R} \) be continuous functions such that \( f(x + 1) = f(x) \) and \( g(x + 1) = g(x) \) for all real numbers \( x \). Prove that
\[ \lim_{n \to \infty} \int_0^1 f(x)g(nx) \, dx = \int_0^1 f(x) \, dx \int_0^1 g(x) \, dx. \]

Putnam B3, 1967