NAME: __________________________ Math 0120 - Business Calculus

Review for MidTerm 1

Answer each of the following. If you wish to receive partial credit, please show all work (NO WORK = NO CREDIT.) In particular, credit will not be given for a “correct answer” when the accompanying work is nonexistent. Please circle your final answers.

1. Find the domain of the following functions: Please leave your answers in interval notation.
   
   (a) \( f(x) = \frac{\sqrt{2x-3}}{x-8} \)
   
   (b) \( g(x) = \ln(\sqrt{x-2}) \)
   
   (c) \( h(x) = \sqrt{\ln(x-2)} \)
   
   (d) \( f(x) = e^{(3x^2+\frac{x}{2}-\pi)} \)
   
   (e) \( f(x) = \begin{cases} -\frac{1}{x+4}, & \text{if } x \leq -1 \\ 3, & \text{if } x > -1 \end{cases} \)

2. Evaluate the following limits. (15 points)
   
   (a) \( \lim_{x \to 5} \frac{x^2-25}{x+5} \)
   
   (b) \( \lim_{x \to 2} \frac{\sqrt{x-2x}}{x+5} \)
   
   (c) \( \lim_{x \to 2} \frac{\sqrt{x}-\sqrt{3}}{x-2} \)
   
   (d) \( \lim_{x \to \frac{1}{3}} \frac{3x^2-16x+5}{3x-1} \)
   
   (e) \( \lim_{x \to \infty} \frac{21x^4+3x^3-\pi}{3x^4-x+2} \)
   
   (f) \( \lim_{x \to \infty} \frac{3x^2-5x+12}{5x^3-x^2+2x-5} \)
   
   (g) \( \lim_{x \to \infty} \frac{5x^3+2x-14}{12x^2-3x+7} \)
   
   (h) \( \lim_{x \to 2^-} \frac{3x+1}{x-2} \)
   
   (i) \( \lim_{x \to -3^+} \frac{x-5}{x+3} \)

3. \( f(x) = \frac{5}{\sqrt{x}} - 3. \)
   
   (a) Find the derivative of \( f(x) \) using the limit definition.
   
   (b) Then evaluate the derivative when \( x = 3 \), then when \( x = \sqrt{2} \).

4. \( f(x) = \sqrt{2x} - 1. \)
   
   (a) Find the derivative of \( f(x) \) using the limit definition.
   
   (b) Then evaluate the derivative when \( x = \sqrt{2} \), then when \( x = 5 \).
5. \( f(x) = (x - 2)^2 + 3. \)
   (a) Find the derivative of \( f(x) \) using the **limit definition**.
   (b) Then evaluate the derivative when \( x = \frac{1}{4} \), then when \( x = -5 \).

6. For the function \( f(x) \) below
   \[
   f(x) = \begin{cases} 
   -\frac{1}{2}x - 4, & \text{if } x \leq -1 \\
   -2, & \text{if } x > -1 
   \end{cases}
   \]
   (a) Draw the graph of \( f(x) \).

Now evaluate the following limits.

(b) \( \lim_{x \to -1^-} f(x) = \)

(c) \( \lim_{x \to -1^+} f(x) = \)

(d) \( \lim_{x \to -1} f(x) = \)

(e) \( \lim_{x \to 2^-} f(x) = \)

(f) \( \lim_{x \to 2^+} f(x) = \)

(g) \( \lim_{x \to 2} f(x) = \)

(h) \( \lim_{x \to \infty} f(x) = \)

(i) \( \lim_{x \to -\infty} f(x) = \)

(j) Is \( f(x) \) continuous at \( x = -1 \)?
   **Explain** your answer.

(k) Is \( f(x) \) continuous at \( x = 2 \)?
   **Explain** your answer.

(l) On what **interval** is \( f(x) \) continuous?
7. Find the first and second derivatives of each of the following functions **clearly labeling both** using the Power Rule.

(a) \( g(x) = 5\sqrt{x} - 9x^3 - \pi \)
(b) \( h(x) = \frac{-x^8 + x^3 + 4}{x^5} \)
(c) \( f(b) = 3^4 \)
(d) \( k(r) = (r - 3)(2r^3 + 1) \)

8. Find the derivative of each of the following functions using **any** rules that you wish. **Do not simplify your answer.**

(a) \( m(x) = 6\sqrt{x} (2x + 1) \)
(b) \( n(x) = \frac{x^3 - 1}{x - 1} \)
(c) \( p(x) = \sqrt[3]{9x - 1} \)

9. Find the equation of the tangent line to the curve \( k(x) = 2\sqrt[3]{x^4} + 7 \) when \( x = 8 \).

***For more questions like this, look on pages 121, 137, and 162 (The odd answers are in the back of the book.)***

10. After \( t \) hours a car is a distance \( s(t) = 60t + \frac{100}{t^2 + 3} \) miles from its starting point.

(a) Find the velocity at 2 hours.
(b) Find the acceleration at 2 hours.

***For more questions like this, look on pages 151 numbers 33 - 39. (The odd answers are in the back of the book.)***

11. A company can produce miniature robots where the cost function is \( c(x) = x^3 - 3x + 34 \).

(a) Find the **marginal cost** function.
(b) Find the **average cost** function.
(c) Find the **marginal average cost** function.
(d) Evaluate the **marginal cost** function when the company has produced 12 miniature robots.
(e) Interpret your answer from part \( d \).

***Conceptual questions to ask yourself***

(a) When will a function have a vertical asymptote?
(b) When we look at the limits approaching vertical asymptotes, what kinds of answers are we looking for?
(c) What are the rules for horizontal asymptotes?
(d) What **three things** do we need to check in order to determine if a function is continuous at a point?
(e) What is the average rate of change? Instantaneous rate of change? Marginal cost ( revenue / profit)? Average cost ( revenue / profit)? Marginal Average cost ( revenue / profit)?
(f) What are the different notations for the derivative?

More practice? Look at the Chapter 2 Review Exercises starting on page 173. The book suggests that a practice test would be the numbers in purple. Remember that you are also supposed to be following along with the practice problems in the book that were handed out with the schedule for this semester.

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