Problem 1. Given any five points on a sphere, show that some four of them must lie on a closed hemisphere.

Problem 2. Let \((x_n)_{n \geq 1}\) be a sequence defined by \(x_1 > 0\) and \(x_{n+1} = \log(1 + x_n)\), \(n \geq 1\). Study the convergence of \(x_n\) and compute \(\lim_{n \to \infty} nx_n\).

Problem 3. Let \(x, y\) be positive real numbers such that \(x^y + y = y^x + x\). Show that \(x + y - xy \leq 1\).

Problem 4. Let \(f : [0, 1] \to \mathbb{R}\) be a continuous and differentiable function on \((0, 1)\). If there exists \(a \in (0, 1]\) such that \(\int_0^a f(x)dx = 0\), then show that
\[
\left| \int_0^1 f(x)dx \right| \leq \frac{1 - a}{2} \sup_{x \in (0,1)} |f'(x)|.
\]

Problem 5. Let \(A_1, A_2, \ldots, A_m \in M_n(\mathbb{C})\) satisfying \(A_1 + A_2 + \ldots + A_m = mI_n\) and \(A_1^2 = A_2^2 = \ldots, A_m^2 = I_n\). Show that \(A_1 = A_2 = \ldots = A_m\).

Problem 6. Let \(a_0 = 1\), \(a_1 = 2\), and \(a_n = 4a_{n-1} - a_{n-2}\) fr \(n \geq 2\). Find an odd prime factor of \(a_{2015}\).