COMBINATORICS HOMEWORK

1. On each face of a regular icosahedron is written a non-negative integer such that the sum of all 20 integers is 39. Show that there are two faces that share a vertex and have the same integer written on them.

2. Let $d_1, \ldots, d_{12}$ be real numbers in the open interval $(1, 12)$. Show that there exist distinct indices $i, j, k$ such that $d_i, d_j, d_k$ are the side lengths of an acute triangle.

3. There are 2010 boxes labelled $B_1, \ldots, B_{2010}$ and $2010 \cdot n$ balls have been distributed among them, for some positive integer $n$. You may redistribute the balls by a sequence of moves, each of which consists of choosing an $i$ and moving exactly $i$ balls from box $B_i$ into any one other box. For which values of $n$ is it possible to reach the distribution of exactly $n$ balls in each box, regardless of the initial distribution?

4. Consider a polyhedron with at least 5 faces such that exactly 3 edges emerge from each vertex. Two players are playing the following game: Each player, in turn, signs his name on a previously unsigned face. The winner is the player who first signs three faces that share a common vertex. Show that the player who signs first will always win by playing as well as possible.