Problem 1. Let $ABCD$ be a convex quadrilateral and $AC \cap BD = \{O\}$. Show that $ABCD$ is a parallelogram if and only if
\[ \overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OD}. \]

Problem 2. Let $M$ be a point in the plane of triangle $ABC$. Prove that the centroids of the triangles $MAB, MBC$, and $MCA$ respectively form a triangle similar to triangle $ABC$.

Problem 3. Let $ABC$ be a triangle and $AA_1, BB_1, CC_1$ its medians. Show that $AA_1, BB_1, CC_1$ can be the side lengths of a triangle.

Problem 4. Let $L_1$ and $L_2$ be distinct lines in the plane. Prove that $L_1$ and $L_2$ intersect if and only if for every real number $\lambda \neq 0$ every point $P$ not on $L_1$ or $L_2$ there exists points $A_1$ on $L_1$ and $A_2$ on $L_2$ such that $\overrightarrow{PA_2} = \lambda \overrightarrow{PA_1}$.

Problem 5. Find the range of the function $f : \mathbb{R} \to \mathbb{R}$,
\[ f(x) = (\sin x + 1)(\cos x + 1). \]

Problem 6. Prove that
\[ \sec^{2n} x + \csc^{2n} x \geq 2^{n+1}, \]
for all integers $n \geq 0$, and for all $x \in \left(0, \frac{\pi}{2}\right)$.

Problem 7. Let $(G, \cdot)$ be a group such that $(ab)^2 = a^2b^2$, for all $a, b \in G$. Show that $G$ is an abelian group.

Problem 8. Let $(G, \cdot)$ be a group such that $x^2 = e$ for all $x \in G$. Show that $G$ is abelian. Here $e$ is the identity element in $G$.

Problem 9. Let $R$ be a ring with identity with the property that $(xy)^2 = x^2y^2$ for all $x, y \in R$. Show that $R$ is commutative.