INTRODUCTION TO ANALYTIC NUMBER THEORY, QIUZHEN COLLEGE, TSINGHUA UNIVERSITY, SPRING 2023

COURSE SYLLABUS

1. General information

Instructors: Cezar Lupu & Dongsheng Wu

Teaching Assistants-Homework Graders: Cezar Lupu & Dongsheng Wu

Emails: lupucezar@gmail.com (Cezar Lupu), wudongsheng14@mails.ucas.ac.cn (Dongsheng Wu)

Lecture time: Monday (Lecture), 13:30-16:05 PM

- 13:30-14:15 PM
- 14:20-15:05 PM
- 15:20–16:05 PM

Location: 6A103 Teaching Building

Office hours: Make an appointment with the instructors via email.

- Cezar Lupu, email address: lupucezar@gmail.com
- Dongsheng Wu, email address: wudongsheng14@mails.ucas.ac.cn

Credits: 3 credit units

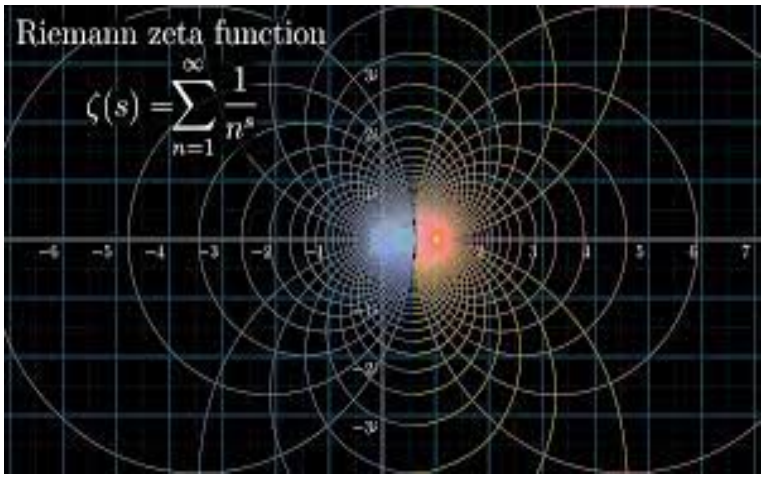
Topics covered: arithmetic and multiplicative functions, Abel summation and Möbius inversion, the Mellin transformation and Perron's formula, Dirichlet series and Euler products, Dirichlet characters, Riemann zeta function, Euler's gamma and beta functions, distribution of prime numbers, prime number theorem, Dirichlet's hyperbola method, Dirichlet's L-functions, primes in arithmetic progression.

2. Course description

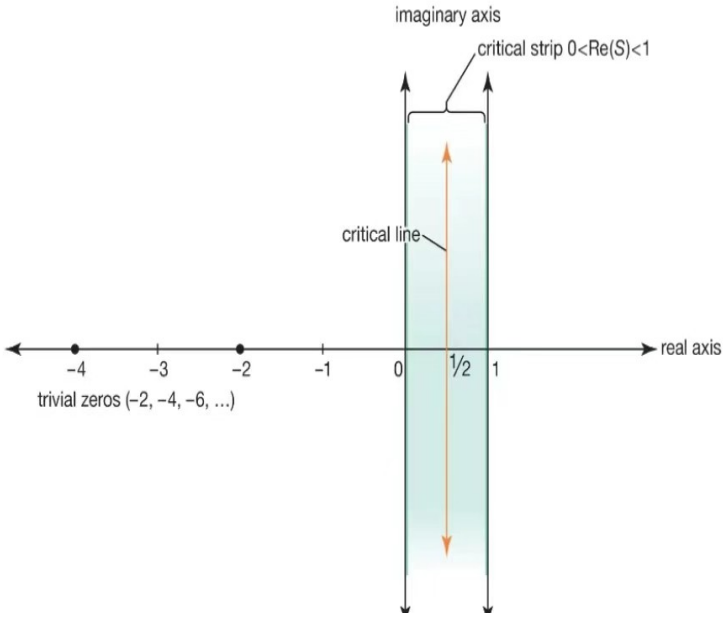
Analytic number theory studies the distribution of prime numbers via mathematical (real & complex) analysis tools. In this course, we study arithmetic functions, but from an analytic perspective, dwelling more on their asymptotic expansion. Moreover, we study the distribution of prime numbers which is still an active research topic today. One of the key figures for this endeavor is the celebrated Riemann zeta function which can be regarded as an important tool in connection with prime numbers. Some of its properties have been studied intensively over the past century. One of the most famous open questions remains the Riemann Hypothesis. Also, another central figure in this course is the prime number theorem which encodes

important information on the distribution of prime numbers. Last but not least, we discuss primes in arithmetic progressions.

By the end of this course, students should develop fundamental knowledge and skills involving basic concepts of the topics covered in this course. Overall, this course will serve as an essential ingredient for further more advanced (graduate level courses) in analysis and number theory.



$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$



Course Outline

- **Chapter 0.** What is analytic number theory? The big picture.

- **Chapter 1.** The fundamental theorem of Arithmetic. A recap of elementary number theory.
- **Chapter 2.** Arithmetical functions and Dirichlet multiplication.
- **Chapter 3.** Asymptotics and averages of arithmetical functions.
- **Chapter 4.** Elementary results on the distribution of primes.
- **Chapter 5.** The prime number theorem. The Riemann zeta function.
- **Chapter 6.** Dirichlet series.
- **Chapter 7.** Primes in arithmetic progression. Dirichlet's theorem.

Course Calendar

- **Lecture 1:** The big picture of analytic number theory (2023/02/20)
- **Lecture 2:** A recap of elementary number theory. (2023/02/27)
- **Lecture 3:** Arithmetic functions and Dirichlet multiplication I (2023/03/06)
- **Lecture 4:** Arithmetic functions and Dirichlet multiplication II (2023/03/13)
- **Lecture 5:** Asymptotics and averages of arithmetic functions I (2023/03/20)
- **Lecture 6:** Asymptotics and averages of arithmetic functions II (2023/03/27)
- **Lecture 7:** Asymptotics and averages of arithmetic functions III (2023/04/3)
- **Lecture 8:** Elementary results on the distribution of primes I (2023/04/10)
- **Lecture 9:** Elementary results on the distribution of primes II (2023/04/17)
- **Lecture 10:** The prime number theorem I (2023/04/24)
- **Lecture 11:** The prime number theorem II (2023/05/1)
- **Lecture 12:** The prime number theorem III (2023/05/8)
- **Lecture 13:** Dirichlet series I (2023/05/15)
- **Lecture 14:** Dirichlet series II (2023/05/22)
- **Lecture 15:** Primes in arithmetic progression I (2023/05/29)
- **Lecture 16:** Primes in arithmetic progression II (2023/06/5)

3. Grading Policy, Grading Scale, Weighted Value of Assignments and Tests

- Homework assignments: **50%** (5 homework assignments in total; you must solve the assigned problems from each homework to get full credit!). There will be around 35-40 problems in total. Solutions must be typed in LaTeXusing Overleaf.
- Midterm (take home) exam: **20%**
- Final (in class) exam: **30%**

Letter grades will then be assigned in accordance with the following correspondence:

- Letter **grade A** = a percentile grade of 90% of higher

- Letter **grade B** = a percentile grade of 80% or higher, that is lower than 90%
- Letter **C**= a percentile grade of 70% or higher, that is lower than 80%
- Letter **D**=a percentile grade lower than 60%

References

[1] T. Apostol, Introduction to Analytic Number Theory, Springer Verlag, 1998.
[2] H.H. Chan, Analytic Number Theory for Undergraduates, World Scientific Publishing Co., 2009.
[3] J-M. De Konnick, F. Luca, Analytic Number Theory, Exploring the Anatomy of Integers, American Mathematical Society Press, 2012.
[4] H. Davenport, H.L. Montgomery, Multiplicative Number theory, Springer Verlag, 2000.
[5] A. Hildebrand, Introduction to Analytic Number Theory, lecture notes, MATH 531 course University of Illinois at Urbana Champaign, 2005.